Numerical Optimization

Homework 1

Due 28.04.2014

Give your answers with logical and/or mathematical explanations. Hand-in your homework in the beginning of a lecture on due date. Late submissions will not be accepted.

1. Let $f_i : \mathbb{R}^n \to \mathbb{R}$ for $i = 1, 2, \ldots, m$ be convex functions. Show that

$$F(x) = \max\{f_1(x), f_2(x), \ldots, f_m(x)\}$$

is also a convex function.

2. Let $\{x_k\}$ and $\{y_k\}$ be two scalar sequences. Show the following using definitions of inf and lim inf:

(a) If $c \leq x_k$ for all k for a scalar c, then $c \leq \inf\{x_k\}$.

(b) $\inf\{x_k\} + \inf\{y_k\} \leq \inf\{x_k + y_k\}$.

(c) $\liminf\{x_k\} + \liminf\{y_k\} \leq \liminf\{x_k + y_k\}$.

3. Let X and Y be two closed sets in R^n. Give an example of X and Y showing that the sum $X + Y := \{x + y : x \in X, y \in Y\}$ is not necessarily a closed set. (You can choose a specific dimension n for your example.)

4. Show that for a convex set C, its closure $\operatorname{cl} C$ is also convex.

5. For $f(x) = x^T H x$, where $x \in \mathbb{R}^n$ and $H \in \mathbb{R}^{n \times n}$, $\nabla^2 f(x) = 2H$ when H is symmetric (in this case, f is twice continuously differentiable). What is $\nabla^2 f(x)$ when H is not symmetric?

6. For $f(x) = \frac{1}{2}\|y - Ax\|_2^2$ where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^k$, and $A \in \mathbb{R}^{k \times n}$, derive the expressions of $\nabla f(x)$ and $\nabla^2 f(x)$ using the chain rule.